A Coxeter diagram is *elliptic* if the corresponding Gram matrix is positive definite.

A connected Coxeter diagram is *parabolic* if the corresponding Gram matrix is positive semi-definite and has corank 1.
A Coxeter diagram is *parabolic* if it is a disjoint union of connected parabolic Coxeter diagrams.

Notation. Let P be a Coxeter polytope in $\mathbb{H}^d$ with facets $f_1, \dots, f_n$.
Let $I \subset \{1, \dots, n\}$ be an index subset.

An *ideal vertex* of P is a vertex lying on the boundary $\partial\mathbb{H}^d$ of $\mathbb{H}^d$.

Combinatorics of a Coxeter polytope P can be read off its Coxeter diagram $\Sigma(P)$:

- **Faces** of P (except for ideal vertices) correspond to elliptic subdiagrams of $\Sigma(P)$:

  $$f = \bigcap_{i \in I} f_i \textit{ is a codimension } |I| \textit{ face of } P$$
  $$\textit{if and only if } \{v_i \mid i \in I\} \textit{ span an elliptic subdiagram of } \Sigma(P).$$

- **Ideal vertices** of P correspond to parabolic subdiagrams of $\Sigma(P)$ of rank $d-1$ (where rank of a Coxeter diagram is the rank of the corresponding Gram matrix).
- P is of finite volume if P is combinatorially equivalent to a Euclidean polytope.

This implies that:

- a compact Coxeter polytope is simple;
- a finite volume Coxeter polytope is simple in edges.

[Vin3] E. B. Vinberg, *Hyperbolic reflection groups*, Russian Math. Surveys 40 (1985), 31–75.